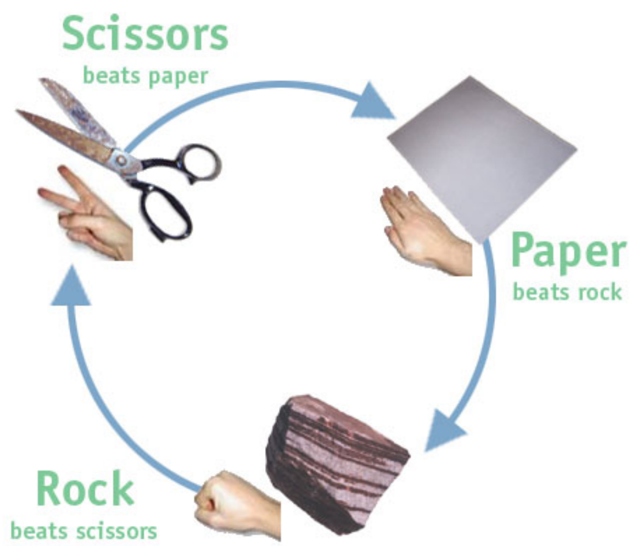
“Rock, Paper, Scissors for Advanced Players”

The game of Rock, Paper, Scissors (RPS) is a hand game played by two players. The players count aloud to three, each time raising one hand in a fist and swinging it down on the count, and upon reaching three changing their hand into one of three shapes signifying either a Rock, Paper, or Scissors. The winner is determined by the following chart (if both players choose the same symbol it is a tie):



The winner scores one point, and the game may be repeated to a predetermined number of points such as 10.

Game Theory says a two-player, *zero-sum* game (no player has an inherent advantage) of incomplete information, like this game, has an optimal solution, known as the Nash Equilibrium (named after mathematician John Nash – subject of the Hollywood movie *A Beautiful Mind* – who first proposed it). Specifically:

If both players have chosen a strategy and neither player can benefit by changing his or her strategy while the other player keep their’s unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.

For the game of RPS, the Nash Equilibrium should be quite obvious. If both players were to randomly choose one of the three shapes, with no predictable pattern, in the long run the score should be even. The optimal solution for the game is to pick randomly. Any other strategy opens yourself up to having your strategy exploited by an observant opponent. For example, if you chose to pick Paper every time, your opponent would eventually learn to pick Scissors every time. (So, with the optimal solution you cannot WIN in the long run, but at least you cannot LOSE either. It’s the best you can do without trying to outguess your opponent.)

QUESTION: What if the rules of the game were changed:

- When one player shows ROCK, and the other shows SCISSORS, the player with ROCK scores **3 points**.
- When one player has SCISSORS, and the other has PAPER, the player with SCISSORS scores **2 points**.
- When one player has PAPER, and the other has ROCK, the player with PAPER scores **1 point**.

Aha! It would be tempting to pick Rock every time now, since it scores the most points, but of course your opponent would figure this out and start picking Paper all the time. So…what is the optimal strategy now? **Hint**: Write three equations representing you or your opponent’s payoffs when you pick each of the 3 objects. Set them all equal to each other to find the unbeatable strategy.

SOLUTION: Let R, P, and S represent the percentage of times you pick Rock, Paper, and Scissors, respectively. And of course:

R + P + S = 100%, so S = 1 - P - R (equation 0)

When our opponent picks Rock, what does he win? Well, expressed in the terms above, he wins:

3(S) – 1(P) = 3(1-P-R) - 1P = 3 – 4P – 3R (equation 1)

When our opponent picks Scissors, what does he win? He wins:

2(P) – 3(R) (equation 2)

When our opponent picks Paper, what does he win? He wins:

1(R) – 2(S) = 1R – 2(1-P-R) = 3R - 2 + 2P (equation 3)

Setting equations 1, 2, and 3 equal (then using equation 0 to find S):

R = 2/6 P = 3/6 S = 1/6

Wow! Who would have known, you don't actually pick Rock the most, you pick Paper the most! It would be easy to implement this strategy by secretly rolling a 6-sided die when your opponent isn't looking, and using that to randomize your choice. Pick in the above proportions and it is guaranteed that no opponent can beat you in the long run!

So why pick paper the most? Well, recall that the optimal strategy must be unbeatable regardless of the strategy used by the opponent. Since a win with rock is so valuable, our only hope at thwarting our opponent's attempt to benefit by picking Rock too often is to pick Paper a large percentage of the time.

Notice that equations 1, 2, and 3 all equal zero when we plug in the values we arrived at for R, P, and S. This is because this is zero sum game. If this wasn't a zero-sum game, all three equations would reduce to a number *n*, which would be the "value" of the game to our opponent. If the number *n* is positive, the game would be skewed in favor of our opponent, and we would be well-advised not to play it.

If we use a non-optimal strategy, game theory says that our opponent can modify his strategy and defeat us. Suppose we randomly chose one of the three shapes each time (R = 1/3, P = 1/3, S = 1/3). Using equation 1, we see that our opponent could always pick Rock and win an average of 3(1/3) – 1 (1/3) = 2/3 of a point per play! Not good for us!